

Drag force of Anisotropic plasma at finite $U(1)$ chemical potential

Long Cheng¹, Xian-Hui Ge¹, Shang-Yu Wu^{2,3}

¹*Department of Physics, Shanghai University, Shanghai 200444, China*

²*Department of Electrophysics, Yau Shing Tung Center, National Chiao Tung University, Hsinchu 300, Taiwan.*

³*National Center for Theoretical Science, Hsinchu, Taiwan.*

physcheng@shu.edu.cn, gexh@shu.edu.cn, loganwu@gmail.com

Abstract

We perform the calculation of drag force acting on a massive quark moving through an anisotropic $\mathcal{N} = 4$ SU(N) Super Yang-Mills plasma in the presence of a $U(1)$ chemical potential. We present the numerical results for any value of anisotropy and arbitrary direction of the quark velocity with respect to the direction of anisotropy. We find the effect of chemical potential or charge density will enhance the drag force for the our charged solution.

1 Introduction

The experimental data in the Relativistic Heavy Ion Collider (RHIC) [1, 2] show that the quark gluon plasma (QGP), as deconfined phase of QCD at high temperature and high number density, is the strongly coupled fluid rather than the weakly coupled gas of quarks and gluons. Thus the perturbative QCD is no longer reliable and we should explore the non-perturbative methods of QCD.

The AdS/CFT correspondence [3–5] provides a powerful tool to investigate the strongly coupled system in condensed matter physics (for reviews, see [6, 7]) like superconductors [8–10], Lifshitz fixed point [11–14], Quantum chromodynamics [15, 16] and heavy ion collisions like photon production [17, 18], elliptic flow [19], drag force [21, 22], jet quenching [23, 24], Langevin coefficients [25] and anomalous transport [27–30]. Using the AdS/CFT correspondence, we can study the strongly $\mathcal{N} = 4$ Super-Yang-Mills plasma through considering IIB supergravity in $AdS_5 \times S^5$. One significant result is the calculation of the ratio of shear viscosity to entropy density of QGP, this ratio is universal and equal to $1/4\pi$ [20], which matches the experimental data very well. This indicates the QGP is a strongly coupled system and the AdS/CFT presents a useful method to investigate the properties of QGP at least at qualitative level.

It is well-known that the most important quantities of QGP are drag force and jet-quenching parameter. In the context of AdS/CFT [21, 22], the moving heavy quark in the thermal medium is dual to a probe open string with infinite mass, which is attached to the boundary of bulk space-time and stretch to the black hole horizon. So the dynamics of string can give us the effects of $\mathcal{N} = 4$ Super-Yang-Mills plasma in which quark is moving. The similar study for jet quenching for $\mathcal{N} = 4$ SYM plasma can be found in [23, 24], where the jet-quenching parameter was obtained by calculating the expectation value of a closed light-like Wilson loop in the dipole approximation.

In this paper, we will study the moving quark in the anisotropic QGP with chemical potential, since the QGP after creation in a short time is anisotropic and real experiments are done at finite baryon potential ¹. Our motivations come from the facts that significant observation of the RHIC and the LHC experiments is that the plasma created is anisotropic and non-equilibrium during the period of time τ_{out} after the collision, i.e. it is locally

¹But the model we consider is static and the origin of anisotropy in this model is different than the QGP anisotropy.

anisotropic at the time $\tau_{out} < \tau < \tau_{iso}$, a configuration to be described by the hydrodynamics with the anisotropic energy-momentum tensor.

Another motivation comes from the fact that in condensed matter physics, some materials are anisotropic, with different properties in different directions. For example, for high temperature cuprates, the crystal structure of such superconductors shows multi-layer of CuO_2 planes with superconductivity taking place between these layers. The electric transport perpendicular to the CuO_2 plane is more difficult than the electric transport in the CuO_2 plane [26]. It is natural to investigate drag force in anisotropic system at finite chemical and compare the results with those isotropic cases.

From the point of view of holography, the anisotropic plasma with chemical potential is dual to a anisotropic charged black brane [31, 32], hence to compute the drag force experienced by an infinitely massive quark, we should consider a string in anisotropic charged black brane. We will show the results analytically and numerically respectively. The discussion for chargeless anisotropic plasma, shear viscosity-entropy density ratio and its energy loss in the framework of AdS/CFT can be found in [18, 34–39].

This paper is organized as follows. In Section 2, we briefly describe the construction of the anisotropic charged black brane solution. In Section 3, we calculate the drag force acting on a massive quark moving through the plasma. Furthermore, in the small anisotropy and high temperature limit, we can perform the calculation of the drag force analytically. In Section 4, we show our numerical results for the prolate anisotropy. In Section 5, we conclude with a brief summary of our results.

Note: When this paper is in the preparation, the paper [40] appeared which has some overlap with this paper. The differences between our paper and [40], are that we consider the drag force for the large charge case in particular.

2 Anisotropic charged black brane solution

The five dimensional axion-dilaton-Maxwell-gravity bulk action reduced from type IIB supergravity is given by [31, 32]

$$S = \frac{1}{2\kappa^2} \left[\int d^5x \sqrt{-g} \left(R + 12 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}e^{2\phi}(\partial\chi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \right) - 2 \int d^4x \sqrt{-\gamma} K \right], \quad (1)$$

where we have set the AdS radius $L = 1$, and $\kappa^2 = 8\pi G = \frac{4\pi^2}{N_c^2}$. The AdS_5 part of ten dimensional anisotropic IIB supergravity solution in Einstein frame is given by [32],

$$ds^2 = \frac{e^{-\frac{1}{2}\phi}}{u^2} \left(-\mathcal{F}\mathcal{B} dt^2 + dx^2 + dy^2 + \mathcal{H}dz^2 + \frac{du^2}{\mathcal{F}} \right), \quad (2)$$

$$A = A_t(u)dt, \quad \text{and} \quad \chi = az, \quad (3)$$

which was obtained by dimensional reduction on S^5 from the ten-dimensional metric. So we have ignored the metric of the S^5 part that will play no role in the following discussions and our treatment is in agreement with ten dimensional framework. Note that the anisotropy is introduced through deforming the SYM theory by a θ -parameter of the form $\theta \propto z$, which acts as an isotropy-breaking external source that forces the system into an anisotropic equilibrium state [34]. The θ -parameter is dual to the type IIB axion χ with the form $\chi = az$ ².

The spacetime is required to be anisotropic but homogenous, so that the functions ϕ , \mathcal{F} , \mathcal{B} and $\mathcal{H} = e^{-\phi}$ should only depend on the radial coordinate u . The electric potential A_t in this metric is expressed as $A_t(u) = -\int_{u_H}^u du Q \sqrt{\mathcal{B}} e^{\frac{3}{4}\phi} u$ from the Maxwell equations, where Q is an integral constant related to the charge density. Note that the electric potential has a contribution from the anisotropy through the dilatonic field ϕ . The horizon is at $u = u_H$ and the boundary is at $u = 0$ respectively. The asymptotic AdS boundary requires $\mathcal{F} = \mathcal{B} = \mathcal{H} = 1$. We can plot the numerical solutions in Figs.1 [32].

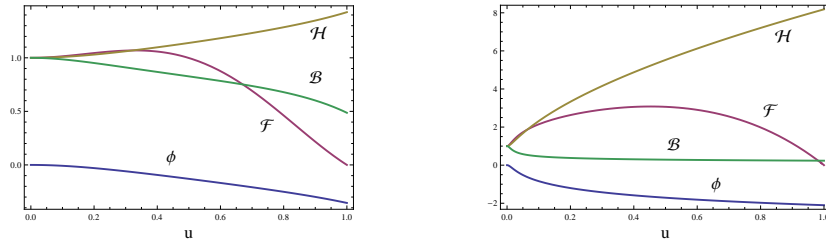


Figure 1: (Color online.) The metric functions for $a = 1.86$, $Q = 6.23$ (left) and $a = 64.06$, $Q = 9.76$ (middle), with $u_H = 1$.

²The model and solutions obtained in the following are the direct generalization of [34, 35].

In the small anisotropy and charge limits, we can obtain the high temperature solution,

$$\begin{aligned}\mathcal{F} &= 1 - \left(\frac{u}{u_H}\right)^4 + \left[\left(\frac{u}{u_H}\right)^6 - \left(\frac{u}{u_H}\right)^4\right]q^2 + a^2\mathcal{F}_2(u) + \mathcal{O}(a^4), \\ \mathcal{B} &= 1 + a^2\mathcal{B}_2(u) + \mathcal{O}(a^4), \\ \mathcal{H} &= e^{-\phi(u)}, \quad \text{with} \quad \phi(u) = a^2\phi_2(u) + \mathcal{O}(a^4),\end{aligned}\tag{4}$$

where $\mathcal{F}_2(u) = \hat{\mathcal{F}}_0(u) + \hat{\mathcal{F}}_2(u)q^2 + \mathcal{O}(q^4)$, $\mathcal{B}_2(u) = \hat{\mathcal{B}}_0(u) + \hat{\mathcal{B}}_2(u)q^2 + \mathcal{O}(q^4)$ and $\phi_2(u) = \hat{\phi}_0(u) + \hat{\phi}_2(u)q^2 + \mathcal{O}(q^4)$ with

$$\begin{aligned}\hat{\mathcal{F}}_0(u) &= \frac{1}{24u_H^2} \left[8u^2(u_H^2 - u^2) - 10u^4 \log 2 + (3u_H^4 + 7u^4) \log \left(1 + \frac{u^2}{u_H^2} \right) \right], \\ \hat{\mathcal{B}}_0(u) &= -\frac{u_H^2}{24} \left[\frac{10u^2}{u_H^2 + u^2} + \log \left(1 + \frac{u^2}{u_H^2} \right) \right], \\ \hat{\phi}_0(u) &= -\frac{u_H^2}{4} \log \left(1 + \frac{u^2}{u_H^2} \right).\end{aligned}\tag{5}$$

and

$$\begin{aligned}\hat{\mathcal{F}}_2(u) &= \frac{1}{24u_H^4(u^2 + u_H^2)} \left[7u^8 + 6u^2u_H^6 + u^4u_H^4(25 \log 2 - 12), \right. \\ &\quad \left. + u^6u_H^2(25 \log 2 - 1) - (u^2 + u_H^2)(12u^6 + 7u^4u_H^2 + 6u_H^6) \log \left(1 + \frac{u^2}{u_H^2} \right) \right], \\ \hat{\mathcal{B}}_2(u) &= \frac{1}{24} \left[-\frac{u^2(11u^4 + 3u^2u_H^2 + 2u_H^4)}{(u^2 + u_H^2)^2} + 2u_H^2 \log \left(1 + \frac{u^2}{u_H^2} \right) \right], \\ \hat{\phi}_2(u) &= \frac{1}{4} \left[-2u^2 + \frac{u^4}{u^2 + u_H^2} + 2u_H^2 \log \left(1 + \frac{u^2}{u_H^2} \right) \right].\end{aligned}\tag{6}$$

where we have used the dimensionless parameter $q = \frac{u_H^3 Q}{2\sqrt{3}}$. Here we would like to justify the usage of the the small anisotropy and charge limits. The analytic solution for the non-perturbative charge was given in [31, 32]. Unfortunately, for the analytic computation of the drag force, it is too difficult to determine some critical parameters.

We can easily obtain the temperature as

$$T = -\frac{\mathcal{F}'(u_H)\sqrt{\mathcal{B}(u_H)}}{4\pi}.\tag{7}$$

Note that the temperature can not be zero unless $a^2 \leq 0$ which corresponds to the oblate anisotropy. In contrast, $a^2 > 0$ corresponds to the prolate anisotropy. Since on the dual quantum field theory side, imaginary a looks like a nonunitary deformation and could lead to a negative field coupling. In this sense, the oblate black brane solution with $a^2 < 0$ could give unphysical results. In the following discussions, we mainly focus on the prolate case.

3 Drag force

On the anisotropic charged brane background, the Nambo-Goto action which governs the dynamics of a probe string in string frame is given by

$$S = -T_0 \int d^2\sigma e^{\phi/2} \sqrt{-\det g_{\alpha\beta}}, \quad (8)$$

where $g_{\alpha\beta} \equiv G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu$ is the induced metric on the two-dimensional world sheet. $X^\mu(\sigma^\alpha)$ are the embedding equations of world sheet in spacetime. In what follows, we generalize the calculations in [37, 38] to charged plasma case and find the additional contribution to the drag force from the chemical potential. We will work in the static gauge, i.e. $\sigma = u$ and $\tau = t$. Since the plasma is anisotropic, we consider the motions of the string in two different directions x and z , which is corresponding to the quark moving at constant velocity v . It is convenient to set the configuration of string as [37]

$$\begin{aligned} x(u, t) &= x(u) \sin \theta + vt \sin \theta \\ z(u, t) &= z(u) \cos \theta + vt \cos \theta. \end{aligned} \quad (9)$$

Obviously θ is the angle between the z -axis and the velocity.

The determinant of $g_{\alpha\beta}$ satisfies

$$-ge^\phi = \frac{-v^2 (\sin^2 \theta + (1 + \mathcal{F} \sin^2 \theta (x' - z')^2) \mathcal{H} \cos^2 \theta) + \mathcal{BF} (1 + \mathcal{F} (x'^2 \sin^2 \theta + \mathcal{H} z'^2 \cos^2 \theta))}{u^4 \mathcal{F}}, \quad (10)$$

so the Lagrangian $\mathcal{L} = -T_0 e^{\phi/2} \sqrt{-g}$ gives the canonical momenta density to the string as

$$\begin{aligned} \Pi_x^\sigma &= \frac{\partial \mathcal{L}}{\partial x'} = -T_0 e^{-\phi/2} \frac{x' \mathcal{BF} + v^2 \mathcal{H} (z' - x') \cos^2 \theta}{u^4 \sqrt{-g}} \sin \theta \\ \Pi_z^\sigma &= \frac{\partial \mathcal{L}}{\partial z'} = -T_0 e^{-\phi/2} \frac{z' \mathcal{BF} + v^2 \mathcal{H} (x' - z') \sin^2 \theta}{u^4 \sqrt{-g}} \cos \theta, \end{aligned} \quad (11)$$

where the prime denotes the derivative with respect to u . Then the equations of motion following from Nambu-Goto action are

$$\begin{aligned} \partial_u \left(-T_0 e^{-\phi/2} \frac{x' \mathcal{BF} + v^2 \mathcal{H} (z' - x') \cos^2 \theta}{u^4 \sqrt{-g}} \sin \theta \right) &= 0 \\ \partial_u \left(-T_0 e^{-\phi/2} \frac{z' \mathcal{BF} + v^2 \mathcal{H} (x' - z') \sin^2 \theta}{u^4 \sqrt{-g}} \cos \theta \right) &= 0. \end{aligned} \quad (12)$$

Note that in string configuration (9), the time part of E.O.M is vanish because of time

independence. Then after integration of (12), we have

$$\begin{aligned} -T_0 e^{-\phi/2} \frac{x' \mathcal{BF} + v^2 \mathcal{H}(z' - x') \cos^2 \theta}{u^4 \sqrt{-g}} \sin \theta &= C, \\ -T_0 e^{-\phi/2} \frac{z' \mathcal{BF} + v^2 \mathcal{H}(x' - z') \sin^2 \theta}{u^4 \sqrt{-g}} \cos \theta &= D, \end{aligned} \quad (13)$$

which involve

$$\begin{aligned} x' &= \frac{e^{\phi/2} u^4 \csc \theta \sqrt{-g} (v^2 (C + D \cot \theta) - C \mathcal{BF} \csc^2 \theta)}{T_0 \mathcal{BF} (\mathcal{BF} \csc^2 \theta - v^2 (1 + \mathcal{H} \cot^2 \theta))}, \\ z' &= \frac{e^{\phi/2} u^4 \csc \theta \sqrt{-g} \sec \theta (v^2 \mathcal{H} (D \csc \theta + C \sec \theta) - D \mathcal{BF} \csc \theta \sec^2 \theta)}{T_0 \mathcal{BF} \mathcal{H} (4 \mathcal{BF} \csc^2 2\theta - v^2 (\mathcal{H} \csc^2 \theta + \sec^2 \theta))}, \end{aligned} \quad (14)$$

where the constant C and D are integral constants corresponding to momenta in two directions. Taking (14) back to (10), we can solve the determinant of induced metric as

$$-ge^\phi = -\frac{2T_0 \mathcal{B} \mathcal{H} (\mathcal{BF} - v^2 (\cos^2 \theta \mathcal{H} + \sin^2 \theta))^2}{I(u)}, \quad (15)$$

with

$$\begin{aligned} I(u) &= u^4 \left[-2T_0^2 \mathcal{B}^2 \mathcal{F}^2 \mathcal{H} + \mathcal{BF} (2D^2 u^4 + \mathcal{H} (2C^2 u^4 + T_0^2 v^2 + T_0^2 v^2 \cos 2\theta (\mathcal{H} - 1) + T_0^2 v^2 \mathcal{H})) \right. \\ &\quad \left. - 2u^4 v^2 \mathcal{H} (D \cos \theta + C \sin \theta)^2 \right]. \end{aligned} \quad (16)$$

The positiveness of the determinant requires [37]

$$\mathcal{B}(u) = \frac{v^2 (\mathcal{H}(u) \cos^2 \theta + \sin^2 \theta)}{\mathcal{F}(u)}, \quad (17)$$

at $u = u_c$ where the denominator $I(u)$ vanishes, which implies that the integral constants satisfy

$$D = C \mathcal{H}(u) \cot \theta, \quad (18)$$

at $u = u_c$. So the determinant of induced metric can be simplified as

$$-ge^\phi = \frac{T_0^2 \mathcal{B} (-\mathcal{BF} + v^2 (\cos^2 \theta \mathcal{H} + \sin^2 \theta))}{-T_0^2 u^4 \mathcal{BF} + C^2 u^8 (1 + \cot^2 \theta \mathcal{H})}, \quad (19)$$

with the choices of

$$C = \pm \frac{T_0 v \sin \theta}{u_c^2}, \quad D = \pm \frac{T_0 \mathcal{H}(u_c) v \cos \theta}{u_c^2}. \quad (20)$$

Then the drag force along the x -direction and z -direction on string are

$$F_x = \frac{T_0 v \sin \theta}{u_c^2}, \quad F_z = \frac{T_0 \mathcal{H}(u_c) v \cos \theta}{u_c^2} \quad (21)$$

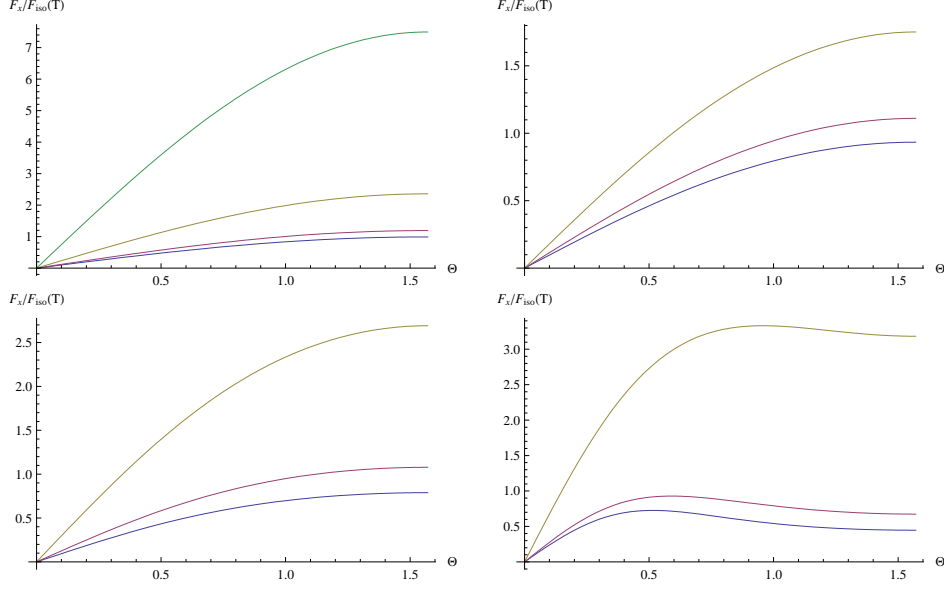


Figure 2: Drag force in x -direction F_x as a function Θ at $v = 0.5$. The four graphs denote $a/T = 1.38$, $a/T = 4.41$, $a/T = 12.2$, $a/T = 86$ respectively, where the colour lines denote $Q = 5$ (Green), $Q = 2$ (Brown), $Q = 1$ (Red), $Q = 0$ (Blue).

4 Numerical analysis

In charged anisotropic plasma, the angle dependence of drag force in x -direction in units of the isotropic drag force are shown in Fig.2 and Fig.3. From the first three plots of Fig.2, we can see that F_x is monotonically increasing functions of Θ when $a \ll T$ and $a \sim T$. However, the last plot of Fig.2 illustrates that F_x is no longer a monotonic function of Θ when $a \gg T$. By contrast, Fig.3 shows that the F_x at $v = 0.9$ is no longer a monotonic function of Θ when $a/T = 12.2$. We also find that for charged plasma, with fixed v , F_x can be larger than $F_{iso} \equiv F_{iso}(q = 0)$ for some interval of Θ which depend on Q and a/T , by contrast, F_x is always smaller than F_{iso} for chargeless anisotropic plasma. Note that the non-monotonic behavior shown in Fig.3 is consistent with the result given in [37], where the authors did not show F_x . This is because if we write down the general expression for F , we can obtain the same ansatz as that given in [37] in the zero-chemical potential limit. Similarly, we can plot the angle dependence of drag force in z -direction in Fig.4 and Fig.5, while F_z is always the monotonically decreasing function of Θ .

The Fig.6 and Fig.7 illustrate the Θ dependence of the drag force F in units of the

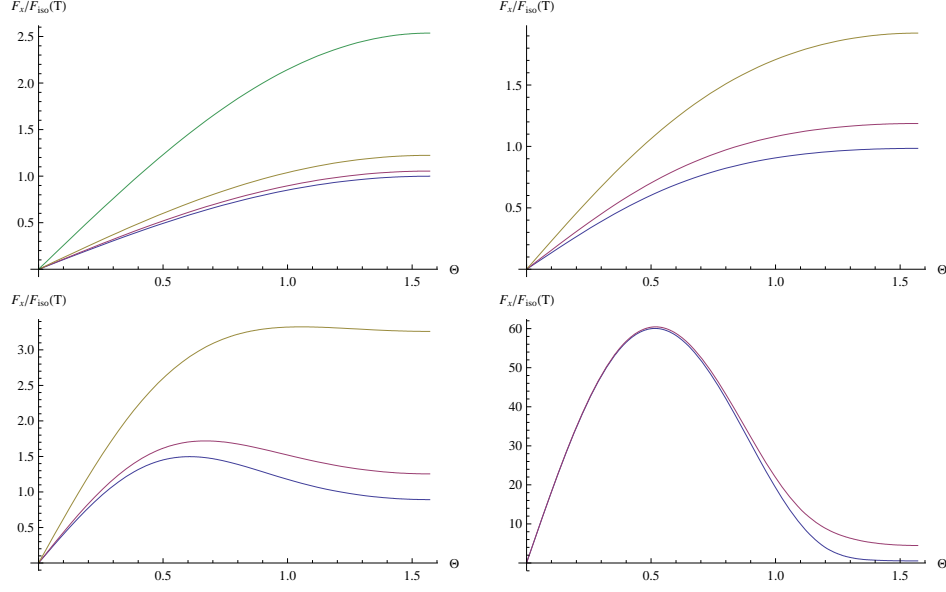


Figure 3: (color online) Drag force in x -direction F_x as a function Θ at $v = 0.9$. The four graphs denote $a/T = 1.38$, $a/T = 4.41$, $a/T = 12.2$, $a/T = 86$ respectively. where the colour lines denote $Q = 5$ (Green), $Q = 2$ (Brown), $Q = 1$ (Red), $Q = 0$ (Blue).

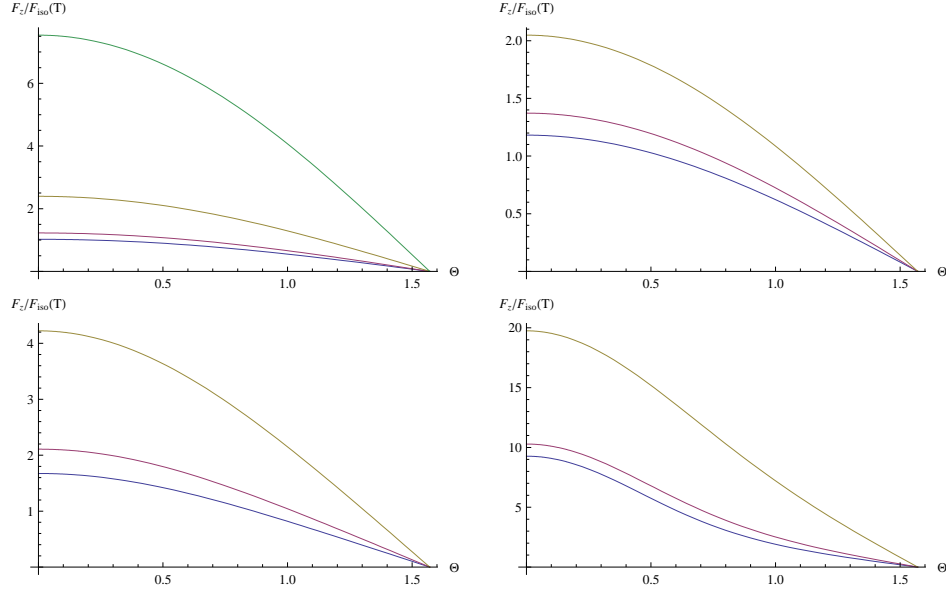


Figure 4: Drag force in z -direction F_z as a function Θ at $v = 0.5$. The four graphs denote $a/T = 1.38$, $a/T = 4.41$, $a/T = 12.2$, $a/T = 86$ respectively, where the colour lines denote $Q = 5$ (Green), $Q = 2$ (Brown), $Q = 1$ (Red), $Q = 0$ (Blue).

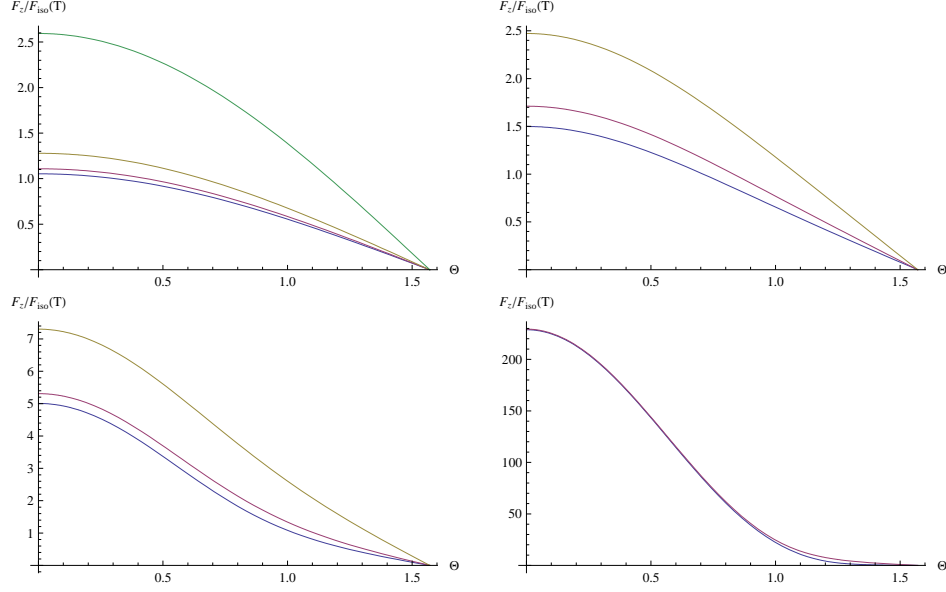


Figure 5: Drag force in z -direction F_z as a function Θ at $v = 0.9$. The four graphs denote $a/T = 1.38$, $a/T = 4.41$, $a/T = 12.2$, $a/T = 86$ respectively, where the colour lines denote $Q = 5$ (Green), $Q = 2$ (Brown), $Q = 1$ (Red), $Q = 0$ (Blue).

isotropic drag force at fixed v and a/T . It is easy to see that F is monotonically decreasing functions of Θ , and when a/T is greater, the falloff of F is faster.

In Fig.8-Fig.11, we can see that the drag force F in units of the isotropic drag force diverges in the ultra-relativistic limit $v \rightarrow 1$ for any $\Theta \neq \pi/2$. We can see that F in charged plasma diverges faster than in chargeless plasma. So the anisotropic drag force is arbitrarily larger than the isotropic case in the ultra-relativistic limit. More interestingly, in the large Q limit, the behavior of drag force F coincides for different angle Θ .

To exhibit the temperature-dependence of drag force more explicitly, we plot the drag force as function of a/T with different Q in Fig.12 and Fig.13. We can see that, unlike anisotropic neutral plasma, in which the drag force in the transverse direction F_x ($\Theta = \pi/2$) is monotonically decreasing function of a/T , F_x in anisotropic charged plasma is no longer a the monotonically function: In the region with $a/T < 1$, F_x increase as a/T increases, but in the region $a/T > 1$, F_x decreases as a/T increases. This means that for non-zero charge Q , the force changes non-monotonically. However, as shown in the plot of the F_z ($\Theta = 0$) in Fig.13, the drag force along the longitudinal direction is always the monotonically increasing function for both neutral plasma and charged plasma.

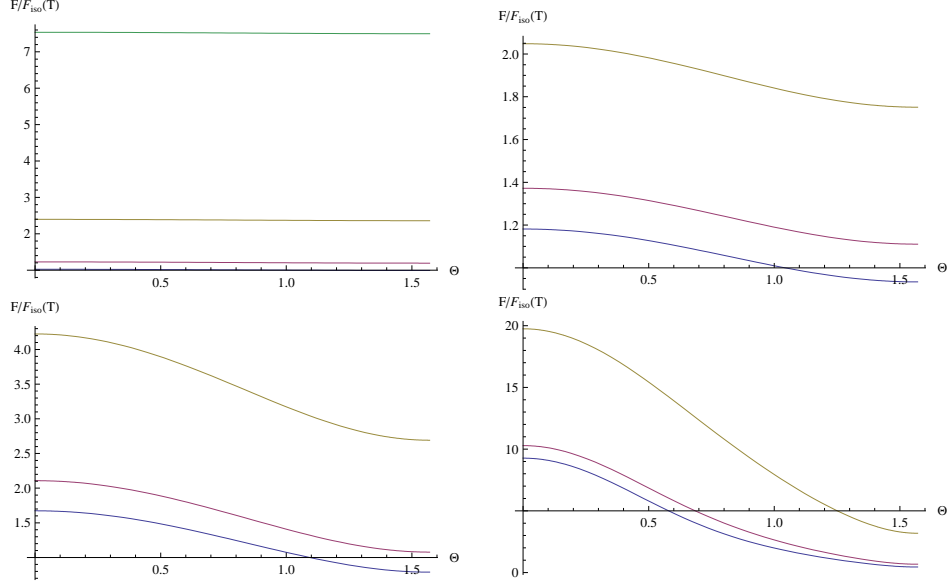


Figure 6: Drag force F as a function Θ at $v = 0.5$. The four graphs denote $a/T = 1.38$, $a/T = 4.41$, $a/T = 12.2$, $a/T = 86$ respectively, where the colour lines denote $Q = 5$ (Green), $Q = 2$ (Brown), $Q = 1$ (Red), $Q = 0$ (Blue).

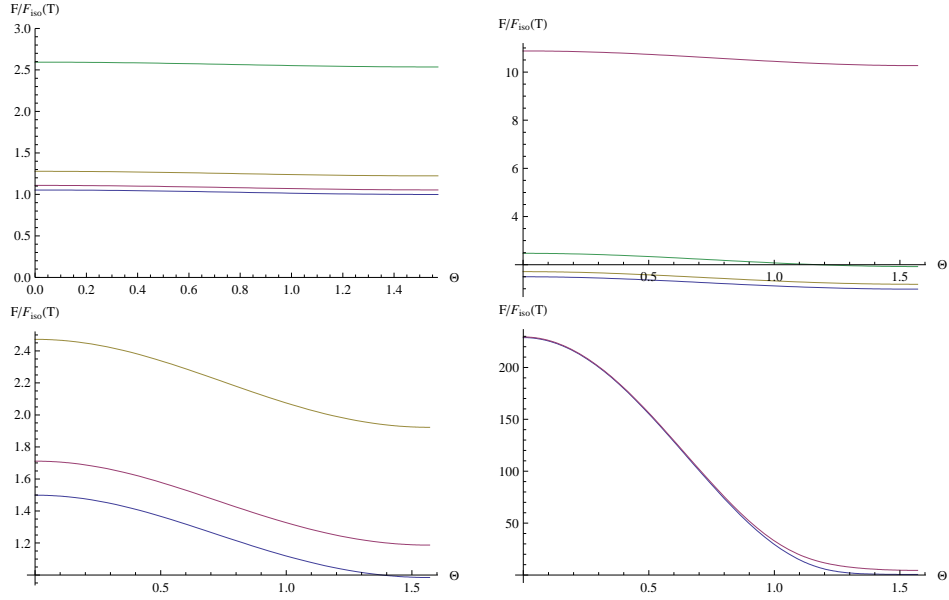


Figure 7: (color online) Drag force F as a function Θ at $v = 0.9$. The four graphs denote $a/T = 1.38$, $a/T = 4.41$, $a/T = 12.2$, $a/T = 86$ respectively, where the colour lines denote $Q = 5$ (Green), $Q = 2$ (Brown), $Q = 1$ (Red), $Q = 0$ (Blue).

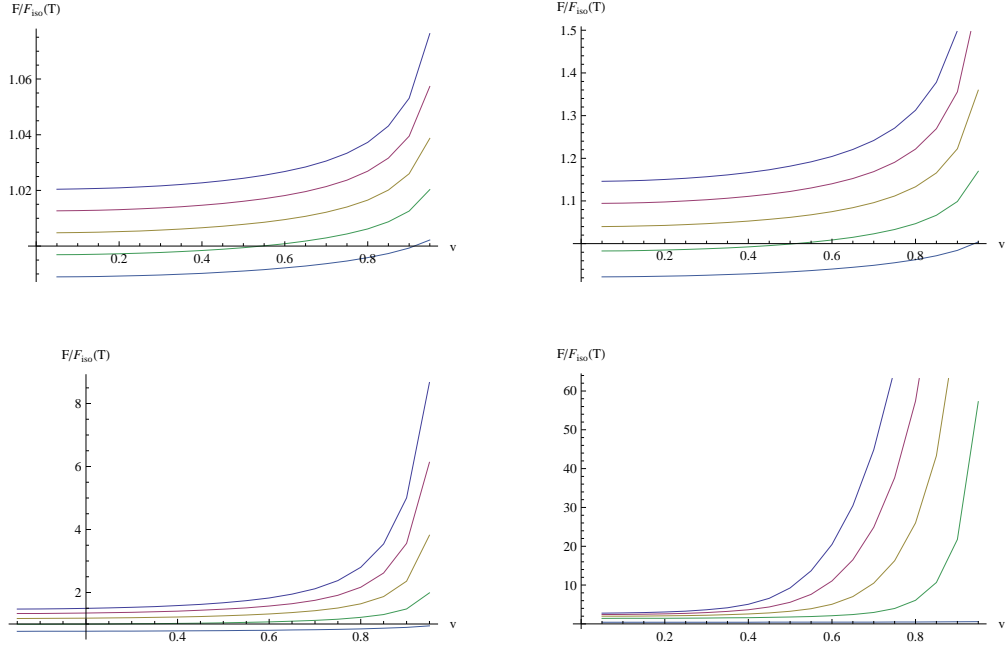


Figure 8: The drag force F as a function of velocity for a quark moving through the plasma with $Q = 0$ (i.e. chargeless). The four graphs correspond to $a/T = 1.38, a/T = 4.42, a/T = 12.2, a/T = 86$ respectively, in which five lines denotes (from the top to down) $\Theta = 0, \Theta = \pi/6, \Theta = \pi/4, \Theta = \pi/3, \Theta = \pi/2$

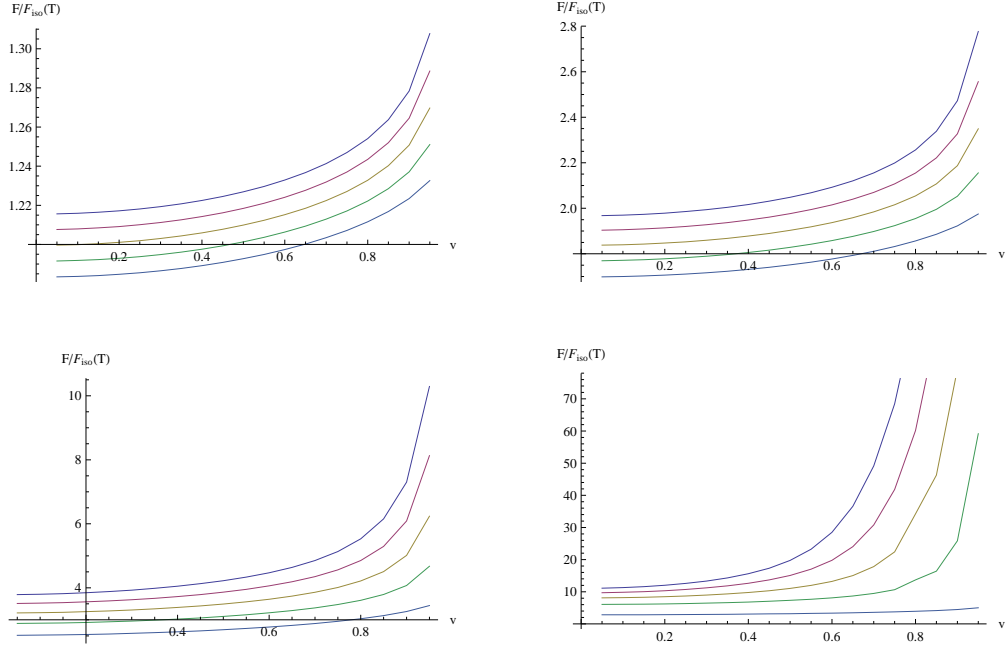


Figure 9: The drag force F as a function of velocity for a quark moving through the plasma with $Q = 2$. The four graphs correspond to $a/T = 1.38, a/T = 4.42, a/T = 12.2, a/T = 86$ respectively, in which five lines denotes (from the top to down) $\Theta = 0, \Theta = \pi/6, \Theta = \pi/4, \Theta = \pi/3, \Theta = \pi/2$

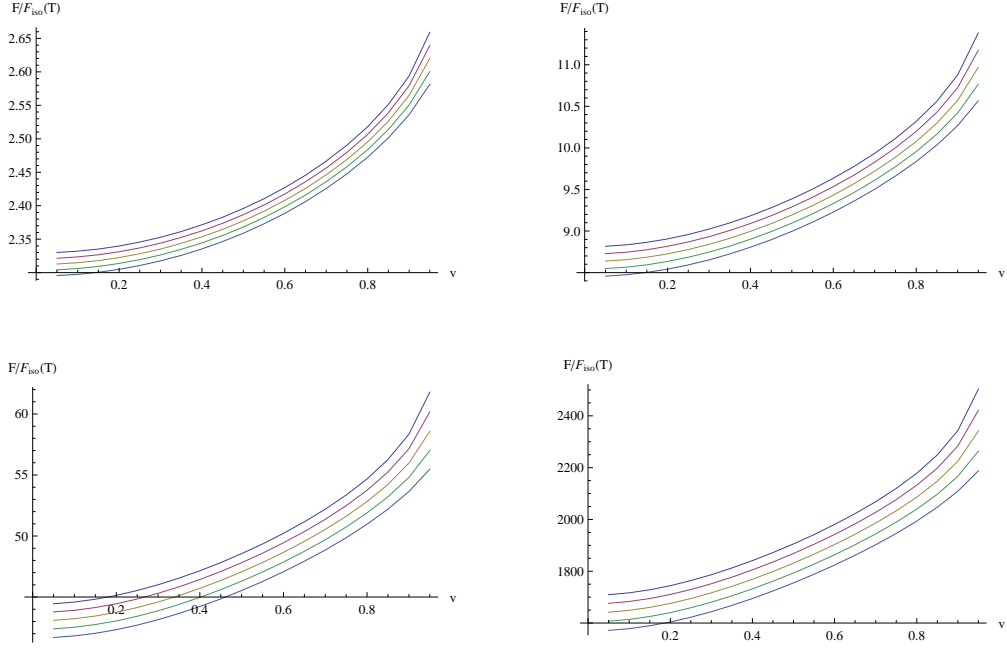


Figure 10: The drag force F as a function of velocity for a quark moving through the plasma with $Q = 5$. The four graphs correspond to $a/T = 1.38, a/T = 4.42, a/T = 12.2, a/T = 86$ respectively, in which five lines denotes (from the top to down) $\Theta = 0, \Theta = \pi/6, \Theta = \pi/4, \Theta = \pi/3, \Theta = \pi/2$

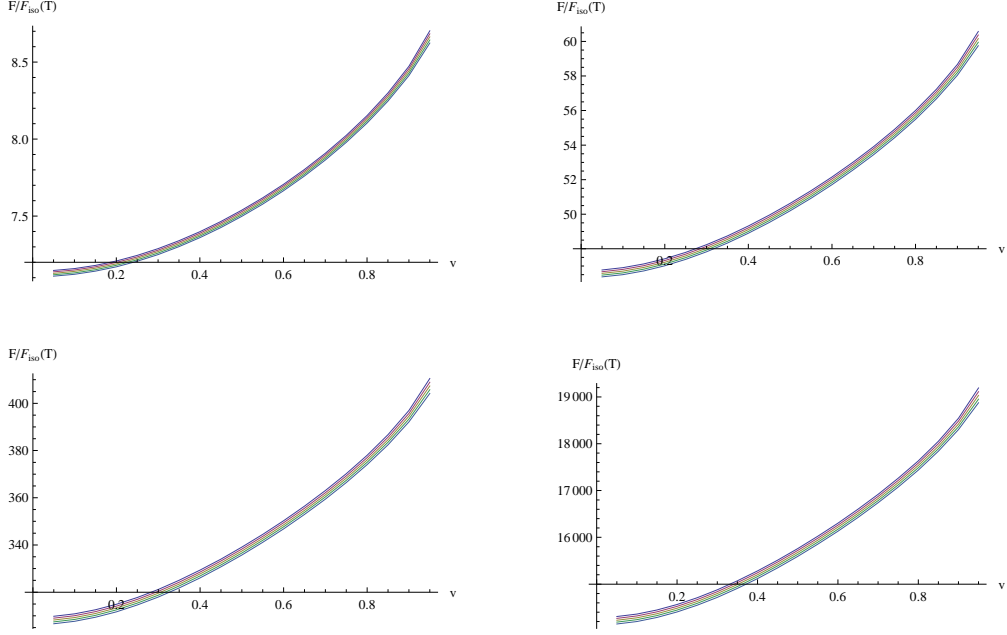


Figure 11: The drag force F as a function of velocity for a quark moving through the plasma with $Q = 10$. The four graphs correspond to $a/T = 1.38, a/T = 4.42, a/T = 12.2, a/T = 86$ respectively, in which five lines denote (from the top to down) $\Theta = 0, \Theta = \pi/6, \Theta = \pi/4, \Theta = \pi/3, \Theta = \pi/2$

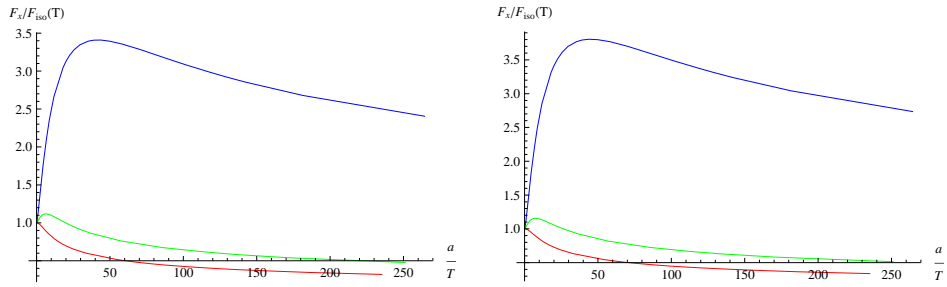


Figure 12: Drag force in x direction as a function a/T at $v = 0.5$ (left) and $v = 0.7$ (right), where the Red, Green, Blue lines represent $Q = 0, Q = 1, Q = 2$ respectively.

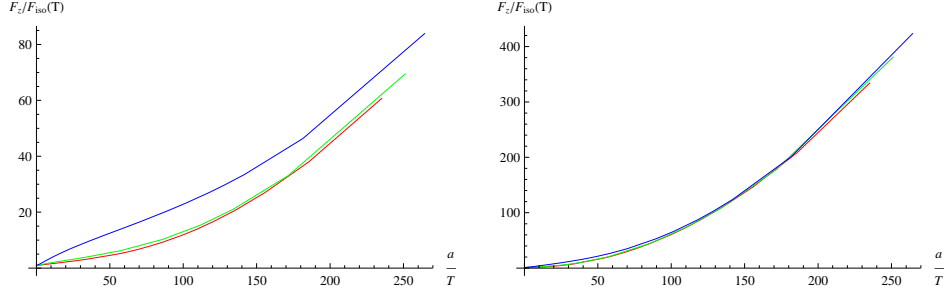


Figure 13: Drag force in z direction as a function a/T at $v = 0.5$ (left) and $v = 0.7$ (right), where the Red, Green, Blue lines represent $Q = 0$, $Q = 1$, $Q = 2$ respectively.

5 Summary

By using AdS/CFT correspondence we have performed the calculations of drag force exerted on a massive quark moving through a charged, anisotropic $\mathcal{N} = 4$ SU(N) Super Yang-Mills plasma. We used the anisotropic charged black brane solution, which is dual to anisotropic QGP with a chemical potential. For a complete study of the drag force in the anisotropic background, we carried out analytic calculation first and obtain some general expressions for the drag force. Different from the isotropic case, where the drag force in charged plasma is always larger than in neutral plasma at the same temperature, for our anisotropic case, we will find that it will be dependent on the explicit value of q and a , which can be seen in the numerical analysis. For arbitrary anisotropy and charge, we presented the numerical results for any prolate anisotropy and arbitrary direction of the quark velocity with respect to the direction of anisotropy. We find the effect of chemical potential or charge density will enhance the drag force for the prolate solution.

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A High temperature analysis

In this appendix, we calculate the drag forces in the limit of high temperature. We consider the high-temperature solution (4). First, plugging the (4) into (17), it is easy to obtain u_c to order of a^2 :

$$u_c = u_0 + u_1 a^2, \quad (22)$$

with

$$\begin{aligned} u_0 &= \left(u_H^2 \sqrt{1-v^2} - \frac{u_H^2(-1+v^2+\sqrt{1-v^2})}{2} q^2 \right)^{1/2}, \\ u_1 &= \frac{u_H^3 \left(1 - \sqrt{1-v^2} - 5 \log 2 + 5 \log(1 + \sqrt{1-v^2}) - v^2(1 - 5 \log 2 + (4 + 3 \cos^2 \theta) \log(1 + \sqrt{1-v^2})) \right)}{48(1-v^2)^{3/4}} \\ &\quad + \frac{u_H^3}{384(1-v^2)^{5/4}(1+\sqrt{1-v^2})} \left(10(1-v^2)(8(1+\sqrt{1-v^2}) \log 2 - v^2(8\sqrt{1-v^2} + (8-7\sqrt{1-v^2}) \log 2)) \right. \\ &\quad + (-80(1+\sqrt{1-v^2}) + v^4(-86+13\sqrt{1-v^2}) + 2v^2(83+38\sqrt{1-v^2})) \log(1+\sqrt{1-v^2}) \\ &\quad \left. + 9v^2 \cos 2\theta(2(v^2-1)(1+\sqrt{1-v^2}) + (v^2(-2+\sqrt{1-v^2}) + 2(1+\sqrt{1-v^2})) \log(1+\sqrt{1-v^2})) \right) q^2. \end{aligned} \quad (23)$$

So, by use of (7) and (21), we are able to derive the drag force

$$\begin{aligned} F_x &= \frac{\pi \sqrt{\lambda} T^2 v}{2} \left(\sqrt{\frac{1}{1-v^2}} - \frac{q^2}{2} \left(1 - \frac{3}{\sqrt{1-v^2}} \right) \right) + \frac{v \sqrt{\lambda} a^2}{48\pi} \left(\frac{1-v^2 + \sqrt{1-v^2} + (4v^2-5) \log(1+\sqrt{1-v^2})}{(1-v^2)^{3/2}} \right. \\ &\quad + \frac{q^2}{2(1-v^2)^2(1+\sqrt{1-v^2})} \left(-(1+\sqrt{1-v^2})(13+30 \log 2) + v^2(23+29\sqrt{1-v^2}) \right. \\ &\quad + 15(4+\sqrt{1-v^2}) \log 2 + v^4(15\sqrt{1-v^2} \log 2 - 2(5+8\sqrt{1-v^2}+15 \log 2)) \\ &\quad \left. \left. + (20(1+\sqrt{1-v^2}) + v^4(17+5\sqrt{1-v^2}) - v^2(37+22\sqrt{1-v^2})) \log(1+\sqrt{1-v^2}) \right) \right), \\ F_z &= \frac{\pi \sqrt{\lambda} T^2 v}{2} \left(\sqrt{\frac{1}{1-v^2}} - \frac{q^2}{2} \left(1 - \frac{3}{\sqrt{1-v^2}} \right) \right) + \frac{v \sqrt{\lambda} a^2}{48\pi} \left(\frac{1-v^2 + \sqrt{1-v^2} + (1+v^2) \log(1+\sqrt{1-v^2})}{(1-v^2)^{3/2}} \right. \\ &\quad + \frac{q^2}{2(v^2-1)^2(1+\sqrt{1-v^2})} \left(v^4(-1-7\sqrt{1-v^2}-15(2-\sqrt{1-v^2}) \log 2) \right. \\ &\quad + v^2(2(-2+\sqrt{1-v^2}) + 15(4+\sqrt{1-v^2}) \log 2) - 5(1+\sqrt{1-v^2})(-1+6 \log 2) \\ &\quad \left. \left. - (v^2(1+v^2)(-2+\sqrt{1-v^2}) + 4(1+\sqrt{1-v^2})) \log(1+\sqrt{1-v^2}) \right) \right), \end{aligned} \quad (24)$$

where we have used $\theta = \pi/2$ and $\theta = 0$, which correspond to x -direction and z -direction respectively. Note that the first part of right hand side of (24) is isotropic force in the

charged plasma F_{iso} :

$$F_{iso}(q) = \frac{\pi\sqrt{\lambda}T^2v}{2} \left(\sqrt{\frac{1}{1-v^2}} - \frac{q^2}{2} \left(1 - \frac{3}{\sqrt{1-v^2}} \right) \right), \quad (25)$$

and, when $q = 0$, we can get the drag forces in the plasma for zero chemical potential, which coincide with the result of [37]. We can see from (25) that, in the isotropic case, the drag force in the charged plasma is always larger than in neutral plasma at the same temperature. In [38], the author obtained a critical velocity $v_c \approx 0.909$ for anisotropic neutral case independent of the temperature and anisotropy. The critical velocity turns out to be charge-dependent and v_c increases as q increases. In the absence of the chemical

q	0	0.04	0.06	0.08	0.1
v_c	0.909	0.912	0.915	0.920	0.926

Table 1: Critical velocity for different q .

potential, equations (24) recover the analytical expressions for the transverse and parallel drag forces given in [38].

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